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IRREVERSIBILITY IN QUANTUM FIELD THEORY[1]

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Abstract

It is shown how the programme of decoherence can be applied in the context of quantum field theory. To illustrate the role of gauge invariance, we first discuss the charge superselection rule in quantum electrodynamics in some detail. We then present an example where macroscopic electromagnetic fields are "measured" through interaction with charges and thereby rendered classical.

A central role in our understanding of quantum theory as a physical theory is played by the attempt to recover consistently from it the classical appearance of our world. Assuming that quantum theory is universally valid, a straightforward application of the superposition principle leads to the occurrence of many superposed classical worlds (i.e., many macroscopic different components of the total wave function), in striking contrast to our everyday experience of *one* classical world. This apparent contradiction has motivated von Neumann more than sixty years ago to impose by hand a phenomenological law *in addition to* the well-understood unitary time evolution of quantum states – the by now infamous "collapse of the wave function". Up to quite recently, this additional rule was indeed only applied as an ad hoc prescription which works for all practical purposes but which lacks any explanation in terms of some fundamental law. Recently, however, there have been suggestions to "put the collapse into the equations, not just the talk" [1]. Typically, such explicit dynamical

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collapse models are of a stochastic nature and lead to the irreversible emergence of "events".

While most contributions to this conference can be, more or less, adjoined to an approach of this kind, I shall here pursue a different route which does not necessarily have to invoke some collapse mechanism and which has also aroused much interest in the past decade – the attempt to understand the irreversible emergence of classical behaviour through interaction with the environment ("decoherence"), see, for example [2, 3, 4], and the references therein. It is the purpose of my contribution to report on some recent applications of decoherence in a field-theoretic context [5, 6]. This presents some novel features over and above those already present in quantum mechanical systems, to which most discussions have been restricted up to now. These novel features are not only concerned with the much more sophisticated technical nature of quantum field theories, but also with conceptual aspects related to the presence of gauge invariance (and, in general relativity, diffeomorphism invariance). After a brief introduction into the general aspects of decoherence, I shall thus present a discussion of the connection between symmetries and superselection rules in quantum field theory and use as an illustration the case of the charge superselection rule in QED. I will then proceed to discuss an example where macroscopic field strengths decohere through their interaction with charges.

The basic observation for the understanding of decoherence is provided by the fact that macroscopic systems cannot be considered, not even approximately, as being isolated from their natural environment [7]. In fact, they are strongly quantum correlated with it. Traditional discussions of the measurement process consider a quantum mechanical system, $\mathcal{S}$ (described by a basis of states $\{\varphi_n\}$), coupled to an "apparatus", $\mathcal{A}$ (described by a basis $\{\Phi_k\}$). In the well-known example by Hepp [8], $\mathcal{A}$ consists of an infinite chain of spin 1/2 particles. A measurement is there considered as complete only in an (unphysical) limit of infinite time, and only with respect to an a priori choice of local observables, see the criticism in [9] and [10].

Taking now into account the natural environment, $\mathcal{E}$ (described by a basis $\{\mathcal{E}_l\}$), of the apparatus, phase relations between different states of the apparatus become delocalised through correlations with the *huge* number of environmental degrees of freedom (photons, air molecules, ...). Tracing them out in the total, quantum-entangled, state (I consider the simplest case of a correlation)

$$|\Psi\rangle = \sum_n c_n|\varphi_n\rangle \otimes |\Phi_n\rangle \otimes |\mathcal{E}_n\rangle \tag{1}$$

leads to a reduced density matrix for $\mathcal{A}$ of the form

$$\begin{aligned}
\rho_{\mathcal{A}} &= \mathrm{Trace}_{\mathcal{E}}|\Psi\rangle\langle\Psi| \\
&= \sum_{n,m} c_n^* c_m |\varphi_m\rangle \otimes |\Phi_m\rangle\langle\mathcal{E}_n|\mathcal{E}_m\rangle\langle\varphi_n| \otimes \langle\Phi_n| \\
&\approx \sum_n |c_n|^2|\varphi_n\rangle \otimes |\Phi_n\rangle\langle\Phi_n| \otimes \langle\varphi_n|,
\end{aligned} \tag{2}$$

where the last step follows from the approximate orthogonality of different environmental states (which is what happens in realistic cases). Thus, the density matrix (2) assumes the form of an *approximate* ensemble, and it seems *as if* the system has "collapsed" into one of the states φ_n with a probability $|c_n|^2$.

If initially there is no (or almost no) quantum entanglement between $\mathcal{A}$ and $\mathcal{E}$, the local entropy

$$S = -k_B \mathrm{Tr}(\rho_{\mathcal{A}} \ln \rho_{\mathcal{A}}) \tag{3}$$

will increase by this interaction – classical properties emerge in a practically *irreversible* manner, since in realistic cases the environmental degrees of freedom never return to their initial state because of the enormous Poincaré times usually involved.

I must emphasise that the result (2) does not yet imply the observation of a definite measurement outcome – only the interference terms have locally disappeared, and the total state (1) is still a pure state. To explain the occurrence of *one* measurement result, one must adhere to one of the following options. The first possibility is that the total state is *really* given by (1). In the framework of this "many-worlds interpretation" facts emerge only through the locality of observers who have only a very restricted algebra of observables at hand. The second possibility has to invoke an explicit collapse mechanism for the total state in the sense mentioned at the beginning [1]. A decision between these two options cannot yet be made and is to a large extent a matter of taste [11]. It is, however, important to keep in mind that it is in principle possible to distinguish between these options, since recoherence would only be possible in the first case. In fact, it has drastic consequences for the arrow of time in a recollapsing quantum universe [12].

Let me now turn to QED and the charge superselection rule [6]. This may also serve as a prototype for other gauge theories, which are not discussed here.

Consider first the classical theory. Infinitesimal gauge transformations parametrised by an arbitrary function $\xi(\mathbf{x})$ are generated by

$$Q^{\xi} = \int d^3x (E^a \partial_a \xi + \rho \xi), \tag{4}$$

where E^a denotes the components of the electric field strength, and ρ is the charge density. Integration by parts yields

$$Q^{\xi} = \int_{S_\infty} d\sigma n_a E^a \xi - \int d^3x \; \xi (\partial_a E^a - \rho). \tag{5}$$

The surface integral is over S_∞, the "sphere at infinity", and n_a is the outward pointing normal. An important feature in electrodynamics, which is connected with the presence of gauge symmetry, is the Gauss constraint equation,

$$\mathcal{G} \equiv \partial_a E^a - \rho = 0. \tag{6}$$

Consequently, on the constraint surface,

$$Q^{\xi}|_{\mathcal{G}=0} = \xi \int_{S_\infty} d\sigma n_a E^a \equiv \xi \; Q, \tag{7}$$

where Q denotes the total charge. It is an observable in the formal sense, since it commutes with $\mathcal{G}$ on the constraint surface.

In the quantum theory the above relations remain formally valid as operator equations. The charge ρ is then given by $-ie\pi_\psi\psi$, where ψ is the spinor field, and π_ψ its conjugate momentum. If quantisation is performed in the functional Schrödinger picture [13], the constraint (6) is implemented as a restriction on physically allowed wave functionals, $\Psi[A_a, \psi]$, as

$$\partial_a \frac{1}{i}\frac{\delta\Psi}{\delta A_a} = -ie\psi\frac{\delta\Psi}{\delta\psi}. \tag{8}$$

This equation expresses the simultaneous invariance of the wave functional with respect to local gauge transformations of the vector potential and the spinor field.

The Gauss constraint only generates *asymptotically trivial* gauge transformations, as can be seen from (5). How should one interpret the remaining gauge transformations? This poses the question on the physical meaning of S_∞ and, thus, the role of infinity in this discussion. One can distinguish between two possibilities. First, S_∞ may lie outside a quantum mechanically closed universe, which means that (although space itself may be finite or infinite) there are no degrees of freedom outside the sphere. In this case Q^ξ should generate redundancy transformations and thus only allow an eigenvalue $Q = 0$ of the total charge operator. In cosmology, this would be a sensible result!

In the second possibility, S_∞ lies far away for all practical purposes, but there may still be charges and/or fields outside. In this case S_∞ may serve as a reference system (compare [14]), and Q^ξ should generate meaningful *symmetries*. The total state may be in a charge eigenstate or not. If it is, for example, in a superposition of two states with negative and positive elementary charges, respectively, $\Psi \equiv \Psi_+ + \Psi_-$, the action of the charge operator would be as follows,

$$e^{i\hat{Q}}\Psi = e^{ie\xi_\infty}\Psi_+ + e^{-ie\xi_\infty}\psi_-, \tag{9}$$

where ξ_∞ is the value of the function $\xi(\mathbf{x})$ at "infinity". An example of an observable (i.e., of a self-adjoint operator which commutes with the Gauss constraint and thus is invariant under local gauge transformations) which has non-vanishing matrix-elements between both charge "sectors", is the Mandelstam observable $\exp(ie\int_{-\infty}^{\mathbf{x}} \mathbf{A}d\mathbf{s})$. Any (quasi-) local observable, however, *commutes* with the total charge, cf. (7), where only S_∞ is involved, since it only has support *inside* the "sphere at infinity". This fact is often referred to as the *charge superselection rule* [15]. Locally, the state is *indistinguishable* from a mixture of states, although the total state may be pure (see [16] for a recent discussion of this in the framework of consistent histories). Due to Gauss law, charges have always been "measured" by the asymptotic fields and are thus always "decohered" with respect to bounded subsystems.

In algebraic field theory one often considers a special case of the second of the above options, that fields may be outside the sphere S_∞, but no sources. For such an "island universe" one can consistently restrict attention to one decohered component of a superposition like (9).

Thus, it is physically more relevant to discuss local superpositions of charges, independent of whether the total state of the Universe is in a charge eigenstate or not [6]. An interesting question is, for example, over what distances an electronic wave packet can be split and *coherently* re-unified. Since the influence of the Coulomb field acts in a reversible manner, the answer depends on the strength of the *irreversible* interaction with the radiation field. Are there quantitative estimates? Joos and Zeh [17] have demonstrated that thermal radiation affects free electrons very efficiently by Thomson scattering. For example, if an initially separating state between electron and field is assumed, there remains (for a temperature $T = 300K$ of the electromagentic field) after one second a coherence length for the electrons of only $0.1cm$ (the dependence of the coherence length on time is as $t^{-1/2}$). Even more effective seems to be the influence of the electron's own radiation field, although there is not yet a definite conclusion about the quantitative outcome [6].

At this point I would only like to mention analogous examples in other theories, such as the mass superselection rule in general relativity, which can be understood along the lines presented here [6].

Due to the mutual interaction of charges and fields, one can not only discuss the measurement of charges by fields, but also the opposite case of a field measurement by charges. It depends of course on the experimental situation, which aspect is the more important one. In fact, a detailed investigation of the field measurement by charges was crucial in the seminal work of Bohr and Rosenfeld.

One may wish to consider, for example, a macroscopic superposition of two electric fields, one pointing upwards, and the other pointing downwards. The total state may be written in the semiclassical form [5]

$$\Psi[\psi\psi^\dagger, A] \approx e^{-iVAE}\chi + e^{iVAE}\chi^*, \qquad (10)$$

where V is the space volume, E and A are the respective components of the electric field and the vector potential, and χ is the state of the electrons. The whole discussion is performed within the functional Schrödinger picture of QED [13]. In the simplest case, χ is assumed to be in a Gaussian state (corresponding to a generalised, A-dependent, vacuum state), but it is straightforward to consider more complicated states. The reduced density matrix for the electric field can be obtained from (10) by tracing out the degrees of freedom corresponding to the electrons, cp. the general expression (2). One finds for the non-diagonal elements of the reduced density matrix (apart from phase factors)

$$\rho_\pm = e^{2iVAE}\mathrm{Trace}_{\psi,\psi^\dagger}\chi^2 \approx e^{2iVAE}\exp\left(-\frac{Ve^2E^2}{512\pi m}\right), \qquad (11)$$

in which the limit $t \gg m/eE$ (which is rapidly reached) was performed. Note that the interaction with the charge states leads to an exponential suppression factor of the corresponding interference terms for the field; in the infrared limit of $V \to \infty$ one finds exact decoherence. In realistic cases, however, a finite coherence width remains, so one can in principle subject these results to experimental confirmation. For an electric field of $E \approx 10^7$ Volts per centimetre, for example, one finds that interference effects are observable on length scales $L \leq 10^{-4}$ centimetres.

Thus, in summary, the programme of decoherence can successfully be applied in the context of quantum field theory, and one can understand the irreversible emergence of classical properties for quantities such as electric charge, mass, or macroscopic field strengths. Of course, a necessary input is the assumption of special initial states of low entropy (i.e., the absence of initial correlations), such that the local entropy (3) for relevant subsystems can increase. This leads eventually into the realm of cosmology and the subject of quantum gravity [12, 18].

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